

Reprise of Foundations 2004

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Abstract

The Foundations 2004 Workshop focused on risk assessment and risk mitigation as they relate to verification, validation, and accreditation. Risk and uncertainty are primarily due to lack of information during decision-making. Therefore, we take decision-making as the context for verification, validation, and accreditation. The state of the art of decision-making is reviewed. The research of Klir and Weirman relating to information and information theories is reviewed, which leads to reviews of uncertainty and risk analysis. The material is illustrated using a simple modeling and simulation example. Recommendations are presented on incorporating decision-making into verification, validation, and accreditation processes.

Keywords: Decision-making, fuzzy arithmetic, interval arithmetic, information.

1 Introduction

The central theme of the Foundations 2004 Workshop on Verification, Validation, and Accreditation (Foundations 2004), held at Arizona State University on 13–15 October 2004 [?], was the role of uncertainty and risk in verification, validation, and accreditation (VV&A). Unlike Foundations 2002, which documented the state of the art in VV&A, Foundations 2004 was a working research meeting with broad representation from academia, Department of Defense (DOD), Department of Energy (DOE) National Laboratories as well as significant foreign participation. The majority of the presentations explored risk, risk assessment, uncertainty and uncertainty quantification. These presentations approached the subject from a standpoint requiring significant statistical expertise; however, many, if not most, VV&A decisions are made by non-statisticians. Hence, there appears to be an education requirement if statistical VV&A processes are to be integrated into system development because of the levels of expertise needed to interpret results. One approach to such education is to adopt a decision-making

model because risk and uncertainty are naturally addressed in that paradigm. A decision-making approach to VV&A can be supported by developing computational libraries for uncertain systems theory, probability, and alternative arithmetics.

The next section reviews the state of the art in decision-making, while Section ?? considers the long history of psychology and mathematics in support of decision-making. Section ?? introduces work by Klir and Weirman relating to information and information theories, and Section ?? reviews uncertainty and risk principles. Section ?? relates the material to the modeling and simulation example. Finally, Section ?? presents the recommendations.

2 VV&A and Decision-Making

While VV&A is often defined in terms of questions to be answered, an alternative approach is to consider it as decision-making processes, allowing for the incorporation of four centuries of research. The most important issues regarding decisions are the frame of reference and the inclusion of the decision-maker's value sets.

Assumptions. Decision-making processes are carried out by **rational** decision-makers. In the modeling and simulation (M&S) context, there are often multiple decision-makers who must share a single **frame of reference (frame)**. This frame of reference, which is primarily terminological, serves as a context for the definition **values** needed to specify a context. In addition, it is assumed that the decision must be made in an **uncertain environment**.

Frame of Reference. Of all VV&A process components, the frame of reference is the least discussed, albeit perhaps the most important. The frame of reference, or

“world view,” involves the implicit assumptions, methods, metaphors, among others, that go into a VV&A decision, providing the context in which it is made. At Foundations 2004, it was evident that there were at least two frames of reference: the Department of Defense and the Department of Energy. Kilikauskas and Hall [?] described the DOD frame, and papers by Logan and Nitta [?] plus the paper by Pilch, Trucano, and Oberkampf [?] described two frames of reference of the National Laboratories. The focus of the DOE workers at the Workshop was, and still is, the nuclear weapons stockpile, primarily a computational science and engineering (CSE) problem, a term denoting modeling and simulation typified by complex physical models. CSE VV&A has an entirely different frame of reference than many DOD VV&A efforts, including fundamental differences in the definitions of basic terms.

Methodological Assumptions.

Since the decision-makers seek to clarify the situation as best they can, it is assumed that they will have goals and objectives that are part of the decision and that they will be able to measure their “levels of satisfaction.” These levels of satisfaction are measure-based values in a particular frame of reference.

3 Decisions

Research into the subject of decisions under uncertain circumstances has a long history, the earliest example being Blaise Pascal and Pierre de Fermat, among others, who, in 1654, considered decisions that arise in the gambling context [?]. Jacob (James) Bernoulli is generally considered the founder of statistics with his *Ars Conjectandi* (1713), while Pierre Simon de LaPlace in *Théorie Analytique des Probabilités* (1812) was the first to consider systematically probability and statistics in science and in other prac-

tical problems. While mathematics can help justify a decision, decision-making is essentially a human activity and, hence, primarily psychological in nature. As a result, decision-making is inherently a modeling exercise; indeed, many non-CSE models arise from the need for more information in the decision-making process.

It has long been established in psychology [?] that human performance in probabilistic inference is suspect. In 1982, Kahneman *et al.* [?] proposed a unified, problem-solving view of decision-making based on John Dewey's problem-solving paradigm:

1. Clearly state the problem using the proper vocabulary.
2. Establish goals, including the constraints and the criteria.
3. Gather relevant information, clearly listing what is known and what is not, addressing cases and/or experiments to decide.
4. Generate possible solutions.
5. Apply the constraints and criteria by conducting thought-experiments or developing prototypes; develop more tests and experiments.
6. Synthesize a solution, testing it as completely as possible.
7. Evaluate and test; review.

This systemization has led to a new understanding of classical decision-making that assumes infinite time and resources, leading Zsombok and Klein [?] to investigate decision-making in the context of short time frames, now known as *naturalistic decision-making*.

Zsombok *et al.* list six parameters for decision-making:

1. Situational awareness
2. Option evaluation
3. Fluidity of the situation
4. Ambiguity of information
5. Stability of goals
6. Time constraints
7. Previous experience

While these are sufficient to categorize common "decision theory" approaches, many standard engineering practices, such as reliability analysis, are decision-making approaches.

4 Information

Fundamental to decision-making are the issues of information, uncertainty, and risk. Lack of information leads to uncertainty. In this context, information is not Shannon information, but *information* as a measure of the ability to distinguish among states. To describe these states, a model of information is needed. Klir and Weirman [?, ?] were the first to describe alternative information models in relation to the alternative forms of uncertainty quantification.

An **information model** is defined as a model in the decision-makers' frame of reference relating the decision model to the uncertainty and risks.

5 Uncertainty and Risk

VV&A are the decision processes for justifying the underlying model in a specific frame, meaning deciding whether or not the model or simulation fits into the knowledge represented within the frame. Uncertainty is an inevitable outcome of attempting to justify new knowledge because some accepted knowledge

in the frame may be at odds or even contradictory to that new knowledge being introduced. Rhetorically, it might be asked which warrants for knowledge are correct and how much of the old knowledge needs to be preserved.

Although Foundations 2004 focused on epistemic (lack of knowledge) and aleatory (fundamental variability) uncertainty, there are others. The judgment literature describes informational uncertainty as the inability to distinguish alternatives from one another. Fuzzy logic introduces lexical uncertainty brought on by inherent vagueness in language.

To develop information models, methods for identifying uncertainty and then quantifying it must be developed. Klir and Weirman describe several such models of uncertainty quantification [?]

- **Bayesian.** The Bayesian (or **subjective** belief) interpretation of probability is that it measures **strength of belief**.
- **Frequentist.** The frequentist interpretation of probability is that it measures chance as the long-run percentage of occurrence of events.
- **Fuzzy.** Fuzzy logic is based on the concept that set membership is not “all or nothing.” The fuzzy community has developed mathematical theories through measure theory with classical theories as subtheories. Fuzzy measure theory includes standard measure theory, which supports axiomatic probability.
- **Possibility Theory.** Possibility theory allows reasoning to be used on imprecise or vague knowledge, making it possible to deal with uncertainties on based this new knowledge.

- **Imprecise Probability.** Imprecise probability is a generic term for the many mathematical or statistical models which measure chance or uncertainty without sharp numerical probabilities. These models include belief functions, Choquet capacities, comparative probability orderings, convex sets of probability measures, fuzzy measures, interval-valued probabilities, possibility measures, plausibility measures, and upper and lower expectations or previsions.

Helton and Oberkampf [?] chaired a workshop in which many of these alternatives were tested against two test problems with very mixed results. Personal discussions with William Oberkampf lead the author to share his conclusions that possibility theory and imprecise probability theory are too undeveloped at the current time for serious VV&A usage. On the other hand, fuzzy simulations have reached a level of sophistication to be applied to engineering problems; see [?]. The Helton and Oberkampf [?] experiment should be pursued.

A comprehensive review of decision-making literature by Zsombok and Klein describes fifteen standard models of decision-making approaches. But those approaches are developed on either Bayesian (subjective) or frequentist accounts. Clearly, extensions to alternative models for decision-making are required.

6 Impact on Modeling and Simulation

Current practice in M&S is to develop a simulation based on a model and then use statistics to validate it. The research proposed here is based on **alternative models of arithmetic** and the use of **uncertain systems**

[?]. By addressing uncertainty from the beginning of the project, the question of validation is naturally developed within the simulation framework. For a complete example, see Chapter Seven of [?].

The principles are in place for such a program: interval arithmetic is supported in both Fortran and C++ by Sun Microsystems. Correct interval arithmetic directly supports fuzzy arithmetic, which can be used to model probability distributions. In addition, there are paradigms for simulations that can include alternative arithmetic or standard arithmetic, such as simulated annealing and discrete event simulations. Although these technologies are well-developed, they are not in the main stream M&S applications, partially due to the lack of exposure to simulationists and partially due to apparent performance issues.

A simple example of these concepts can be illustrated, in the spirit of Helton and Oberkampf, using a sophomore physics problem:

Consider a compact object falling from the top a cliff for which only approximate height data is known. How long will it take for the object to reach the ground?

While the appropriate physics is known,

$$\frac{d^2y}{dt^2} \approx -g + D\left(\frac{dy}{dt}\right)^2. \quad (1)$$

There is a range of values for both g and D ; in fact, it is not even be certain that $D \neq 0$. Compounding the problem is the possible fuzziness of the height data. Assume $D = 0$, $g = [9.7, 9.9]$ meters per second per second and the height as $h = [119.5, 125.0]$ meters. Using Sun's **f95** interval implementation, $t = [4.90, 5.08]$. The interval value for t contains the shortest (4.90) and longest (5.08) times consistent with the input, but it does not indicate the **most likely value**.

To compute the most likely value, we use fuzzy arithmetic. For this example, triangular numbers are used. Let $g = \langle 9.7, 9.8, 9.9 \rangle$ with 9.7 being the low bound, 9.8 being the most likely value, and 9.9 being the high bound. Similarly, let $h = \langle 119.0, 120.0, 125.0 \rangle$. Using this value, $t = \langle 4.90, 4.95, 5.08 \rangle$. Notice that the fuzzy value is within the interval arithmetic value, as it should be.

If $D \neq 0$, then the problem is much more difficult because Equation ?? is solvable for t , but the result is not very intuitive:

$$s = \frac{\log(\cosh(t\sqrt{bg} - a\sqrt{Dg}))}{D}$$

For illustration, the problem is solved instead by using a numerical algorithm. We obtain the fuzzy numbers:

Condition	Height Constant	Drag Coefficient	Time	
Minimum	119	9.7	0.0	4.97
Confident	120	9.8	0.2	5.35
Maximum	125	9.9	0.4	5.74

Notice that there is a 1.5% error in the minimum time between the algebraic answer and the numerical answer. It turns out that the simple expedient of changing the rounding mode does not solve the problem, indicating that different numerical algorithms are needed. This numerical error also illustrates how uncertainty is naturally introduced into the VV&A process: How does the numerical answer relate to the algebraic answer? What are the risks of using one or the other?

7 Conclusions

Foundations 2004 was organized to report on research in risk, uncertainty, and uncertainty quantification. However, a more fundamental result becomes clear: the understanding that

VV&A is ultimately better understood in a decision-making context. Hence, two recommendations are proposed.

Recommendation 1. We recommend that VV&A processes be formulated with decision-making research as a guideline. The fundamentals of decision-making are closely related to problem-solving, as made clear by Kahneman, Solvic, and Tversky. However, the research of Zsombok, Klein, and others have shown that the the problem-solving paradigm may not be used in practice by many decision-makers in certain situations. The psychology of expertise and decision-making continues to be a active area [?, ?]. It is clear that working management with VV&A responsibilities must be able to understand and develop uncertainty models as a natural part of decision-making. Educational programs must be developed to support VV&A practitioners.

We propose that VV&A develop information models based on the standard approaches used in decision-making and based on the problems themselves. For example, CSE models will continue to be evaluated using sophisticated statistical methods developed especially for the problem at hand; we should not expect those methods to work in all situations. There are wealth of tools in the business risk market that bear evaluation.

Recommendation 2. Our second recommendation is that M&S practice focus on modeling and simulation of **uncertain systems**, systems in which uncertainty is a fundamental part of the modeling and simulation design¹. This focus can be accomplished in several ways.

1. Modeling and simulation education must integrate the uncertainty present in models into simulations that can produce information useful in quantifying that uncertainty. Engineering and science courses, traditional CSE courses,

should be modified to include uncertainty issues in models.

2. We must develop trusted software libraries and language platforms that allow simulationists to develop simulations of uncertain systems.
 3. Research must be undertaken to explore alternative forms of uncertainty and uncertainty quantification. Klir and Weirman note that the various types of uncertainty quantification are not fully explored. But the various forms of uncertainty quantification can only be used *after* a model of uncertainty has been developed.
 4. The alternative models of uncertainty that have appeared in the literature are not as developed as the axiomatic probability systems. It may well be too soon to know whether or not these alternative systems will provide new insights. One aspect of the Klir and Weirman approach that has been overlooked is the focus on *information theories*. The author first discussed the decision making paradigm with information focus in [?].
 5. We recommend that an organized effort be made to evaluate these alternative formulations to extend the results of [?].
- We recommend that case studies be developed educate developers on the cost/benefits of VV&A.

We have implemented an extension to Sun's f95 compiler that computes fuzzy functions. We can develop supporting libraries, which will require that the appropriate numerical analysis and numerical methods be explored. This development will be driven by the need for tutorial programs.

¹Laplace, "It is remarkable that a science which began with the consideration of games of chance should have become the most important object of human knowledge." *Théorie Analytique des Probabilités* (1812).

